RANDOM VARIABLE & PROBABILITY DISTRIBUTIONS

Learning Objectives:

- Test from descriptive statistics and probabilities.
- Working with random variables.
- Applications of probability distribution in medical practice.

EXERCISE 1

Many new drugs have been introduced in the last decade to bring hypertension under control to reduce high blood pressure to normal levels. A physician agrees to use a new antihypertensive drug on a trial basis on the first untreated hypertensive patients. From the previous experience with the drug, the drug company expects that for any clinical practice the probability that 0 patients out of 4 will be bought under control to be of 0.008, 1 patients out of 4 will be bought under control to be of 0.076, 2 patients out of 4 to be of 0.265, 3 patients out of 4 to be of 0.411 and the probability that all 4 patients out of 4 to became normotensive to be of 0.240. This probability mass function, or probability distribution, is displayed in the next table:

$$X: \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.008 & 0.076 & 0.265 & 0.411 & 0.240 \end{pmatrix}$$

a. Which is the number of subjects expected to be brought under control for every 4 treated patients?
b. Compute the variance and standard deviation for this random variable.

EXERCISE 2

The probability to have an adverse reaction to a vaccine is equal to 0.15. Which is the probability that 4 out of 6 vaccinated children to have an adverse reaction after receiving the vaccine?

EXERCISE 3

We know that children develop chronic bronchitis in the first year of life in 3 out of 20 household where both parents had chronic bronchitics, as compared with the national incidence rate of chronic bronchitis, which is 5% in the first year of life. Is this difference real or can it attributed to chance? Specifically, how likely are infants in at least 3 out of 20 households to develop chronic bronchitis if the probability of developing disease in any one household is 0.05?

EXERCISE 4

Consider a family with a mother, father and two children.

Let be

A_1 = {mother has influenza}

A_2 = {father has influenza}

A_3 = {first child has influenza}

A_4 = {second child has influenza}

B = {at least one child has influenzae}

C = {at least one parent has influenzae}

D = {at least one person has influenzae}

a. What does $A_1 \cup A_2$ means?
b. What does $A_1 \cap A_2$ means?
c. Are A_3 and A_4 mutually exclusive?
d. What does $A_3 \cup B$ means?
e. What does $A_3 \cap B$ means?
f. Express C in terms of A_1, A_2, A_3, A_4.
g. Express D in term s of B and C.
h. What does $\overline{A_1}$ mean?
i. What does $\overline{A_2}$ mean?
j. Represent $\overline{C}$ in terms of A_1, A_2, A_3, A_4.
k. Represent $\overline{D}$ in terms of B and C.

EXERCISE 5

A drug company is developing a new pregnancy-test kit for use on an outpatient basis. The company uses the pregnancy test on 100 women who are known to be pregnant, of whom 95 are positive using the test. The company uses the pregnancy test on 100 other women who are known to not be pregnant, of whom 99 are negative using the test.

a. What is sensitivity of the test?
b. What is the specificity of the test?
   The company anticipates that of the women who will use the pregnancy test kit, 10% will actually be pregnant.
c. What is the predictive value positive for the test?