Point Estimators & Confidence Intervals

Sorana D. Bolboacă

OBJECTIVES

- Point estimators
- Confidence interval for mean
- Confidence interval for proportion

INFERENTIAL STATISTICS

- Inferential statistics = the process of making guesses about the truth on the population by examining a sample extracted from the population
- Sample statistics = summary measures calculated from data belonging to a sample (e.g. mean, proportion, ratio, correlation coefficient, etc.)
- Population parameter = true value in the population of interest
- Point estimation involves the use of sample data to calculate a single value (known as a statistic) which is to serve as a "best guess" or "best estimate" of an unknown (fixed or random) population parameter.

POINT ESTIMATOR

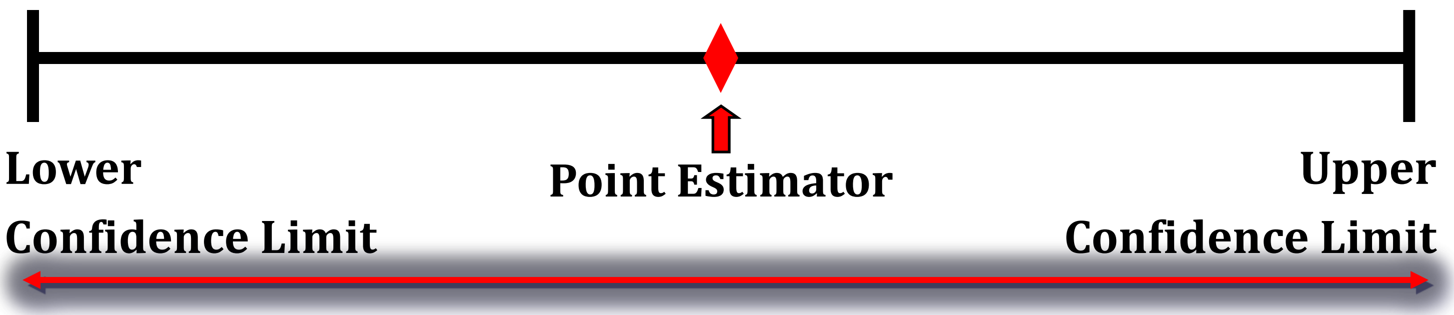
- Point estimation provide one value as an estimate of the population parameter (e.g. the sample mean is a point estimator for population mean)
  - We are interested in the mean of height of 10-years-old boys and girls in the Romania. It would be impossible to measure the height of all 10-years-old boys and girls height so we will investigate a random sample of 30 boys and a random sample of 30 girls of 10-years-old. The sample mean for boys is 140 cm and for girls is 132 cm.
    - The sample mean of 140 cm is a point estimator of boys population mean
    - The sample mean of 132 cm is a point estimator of girls population mean

POINT ESTIMATOR VS. INTERVAL ESTIMATION

- Interval estimation: provide a range of values (an interval) that contain with a high probability the unknown parameter
- Confidence interval: the interval that contain an unknown parameter (such as the population mean) with certain degree of confidence
- It is recommended to estimate a theoretical parameter by using a range of value not a single value
  - It is called confidence interval
  - The estimated parameter belong to the confidence intervals with a high probability.

Point Estimator vs. Interval Estimation

- Point estimator = one value obtained on a sample
  - How much uncertainty is associated with a point estimator of parameter?
- An interval provides more information about a population characteristics that does a point estimator ⟶ confidence interval



Width of confidence interval

INTERVAL ESTIMATION

- **An interval gives a range of values**:
  - Takes into consideration variation in sample statistics from sample to sample
  - Based on observations from 1 sample
  - Gives information about closeness to unknown population parameters
  - Stated in terms of level of confidence. (Can never be 100% confident)
- **The general formula for all confidence intervals is equal to:**

$$\text{Point Estimator} \pm \underbrace{(\text{Critical Value})\times(\text{Standard Error})}_{\textbf{Margin or error}}$$

Table value

INTERVAL ESTIMATION

$$\text{Point Estimator} \pm \underbrace{(\text{Critical Value}) \times (\text{Standard Error})}_{\textbf{Margin or error}}$$

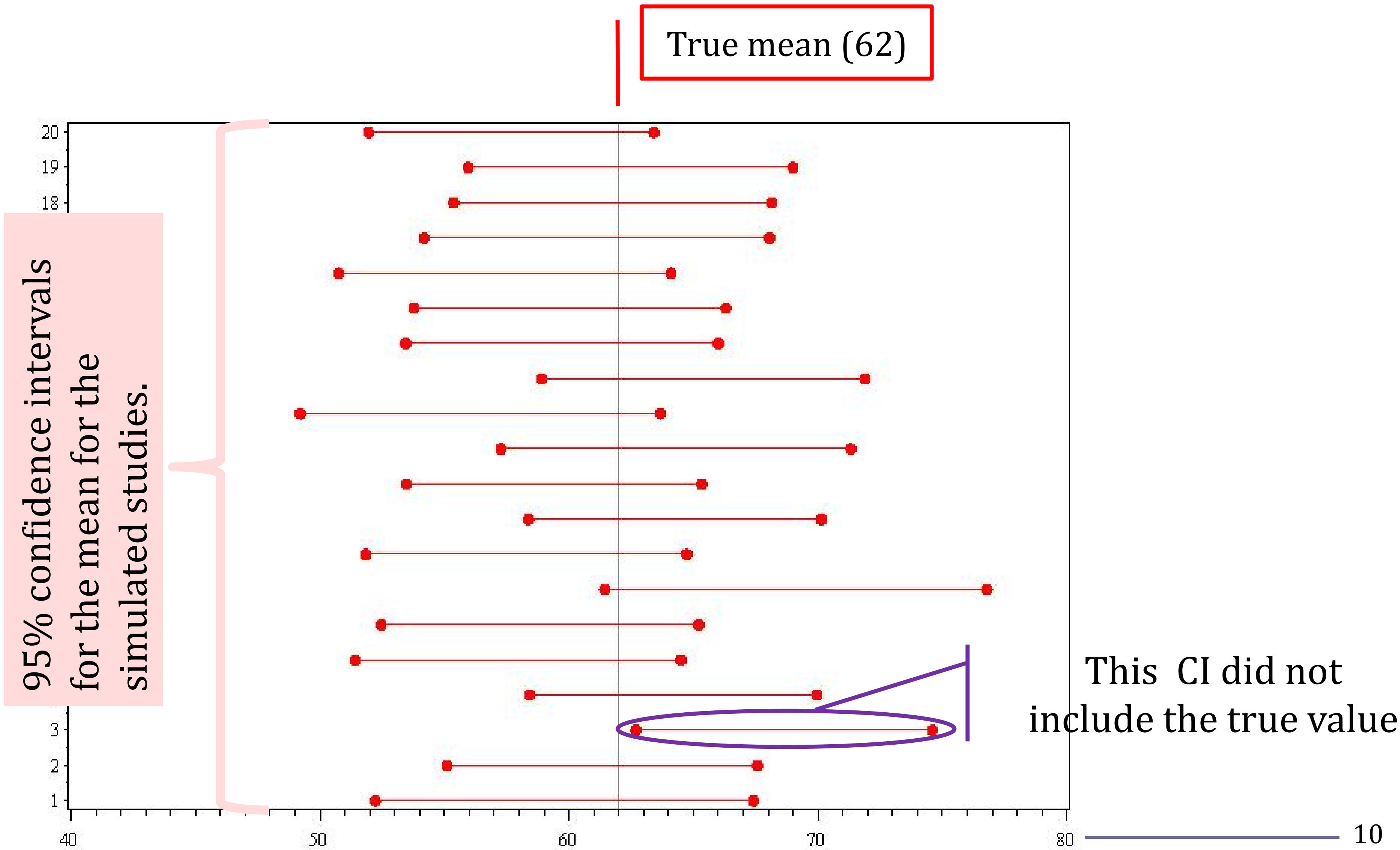
- The margin of error, and hence the width of the interval, gets smaller the as the sample size increases.
- The margin of error, and hence the width of the interval, increases and decreases with the confidence.

INTERVAL ESTIMATION

- Significance level $\alpha = 5\% \rightarrow 95\%$ confidence interval (CI)
- $CI = (1 - \alpha) = 0.95$

- Interpretation:
  - If all possible samples of size *n* are extracted from the population and their means and intervals are estimated, 95% of all the intervals will include the **true value of the unknown parameter**
  - A specific interval either will contain or will not contain the true parameter (due to the 5% risk)

INTERVAL ESTIMATION



CONFIDENCE INTERVALS

- Provides:
  - A plausible range of values for a population parameter.
  - The precision of an point estimator.
    - When sampling variability is high, the confidence interval will be wide to reflect the uncertainty of the observation.
  - Statistical significance.
    - If the 95% CI does not cross the null value, it is significant at 0.05.

CONFIDENCE INTERVALS

- Are calculated taking into consideration:
  - The sample or population size
  - The type of investigated variable (qualitative OR quantitative)

- Formula of calculus comprised two parts:
  - One estimator of the quality of sample based on which the population estimator was computed (standard error)
    - Standard error: is a measure of how good our best guess is.
      - Standard error: the bigger the sample, the smaller the standard error.
      - Standard error: is always smaller than the standard deviation
  - Degree of confidence (standard values)

Confidence Intervals for Means

- Assumptions:
  - Population standard deviation (σ) is known
  - Population is normally distributed
  - If population is not normal, use large sample

$$\left[\overline{X} - Z_{\alpha}\frac{\sigma}{\sqrt{n}}; \overline{X} + Z_{\alpha}\frac{\sigma}{\sqrt{n}}\right]$$

where Z is the normal distribution's critical value for a probability of $\alpha/2$ in each tail

CONFIDENCE INTERVALS FOR MEANS

- Under the normality assumption:

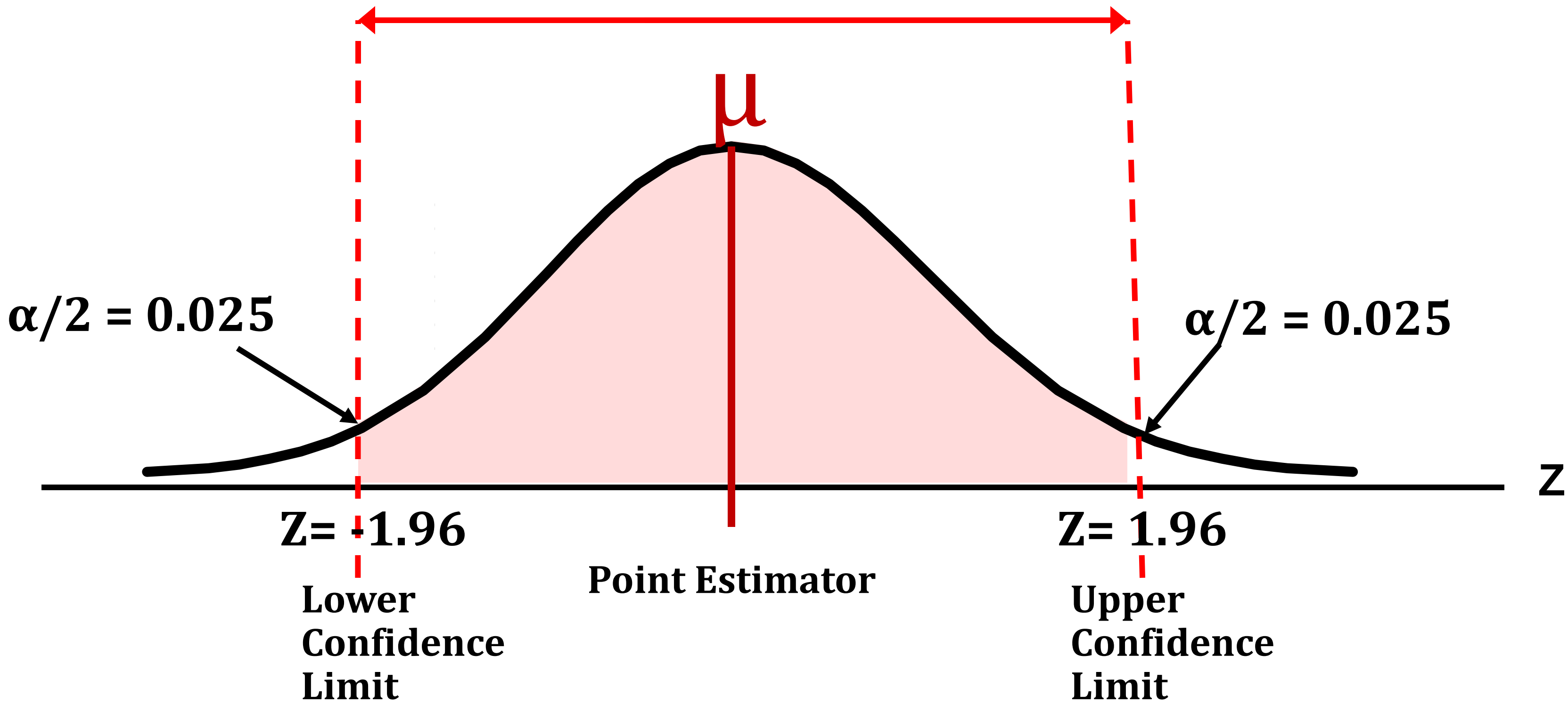
$$P\left(\bar{X} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)=0.95$$

- 95%CI for population mean when standard deviation of the mean is known is:

$$\left[\bar{X} - 1.96\frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96\frac{\sigma}{\sqrt{n}}\right]$$

- In repeated sampling from a normally distributed population with an known standard deviation, 100*(1-α) percent of all intervals of the form above will in the long runs cover the population mean

- Consider a 95% confidence interval:
- $1-\alpha = 0.95$ & $\alpha = 0.05$ & $\alpha/2 = 0.025$



Confidence Intervals for Means

- Consider the distribution of serum cholesterol levels for all female Romanian who are hypertensive and overweight. This population has an unknown mean (μ) and a standard deviation (σ) of 30 mg/dl. We extracted from this population a random sample of 20 subjects and we found a mean of serum cholesterol level ($\bar{X}$) equal with 220 mg/dl.
  - $\bar{X}$ = 220 mg/dl is a point estimator of the unknown mean serum cholesterol level (μ) in the population
  - Because of the sampling variability, it is important to construct the interval able to take into account the sampling variability:

$$95\%\text{CI} = \left(220 - 1.96\frac{30}{\sqrt{20}}, 220 + 1.96\frac{30}{\sqrt{20}}\right) = (207, 233)$$

Length = 233-207 = 26

$$99\%\text{CI} = \left(220 - 2.58\frac{30}{\sqrt{20}}, 220 + 2.58\frac{30}{\sqrt{20}}\right) = (203, 237)$$

Length = 237-203 = 34

Confidence Interval by Examples

Let us suppose that there are 65 country and imported beer brands in the Romanian market. We have collected 2 different samples of 20 brands and gathered information about the price of a 6-pack, the calories, and the percent of alcohol content for each brand. Further, we know the population standard deviation (σ) of price is €1.15. Here are the samples' information:

Sample A: m_A= €4.90, s_A= €1.09

Sample B: m_B= €5.20, s_B= €0.98

Provide 95% confidence interval **estimates of population mean price** using the two samples.

Confidence Interval by Examples

- Interpretation of the results from
  - Sample A: We are **95%** confident that the true mean price is between €4.47 and €5.33
  - Sample B: We are **95%** confident that the true mean price is between \$4.82 and \$5.58
- After the fact, I am informing you know that the population mean was €4.50. Which one of the results hold?
  - Although the true mean may or may not be in this interval, 95% of intervals formed in this manner will contain the true mean.

CONFIDENCE INTERVALS FOR MEANS

- Unknown population mean (μ) & unknown population standard deviation (σ): student t-distribution with n-1 degree of freedom will be used

$$P\left(\overline{X} - t_{\alpha/2}\frac{s}{\sqrt{n}} \leq \mu \leq \overline{X} + t_{\alpha/2}\frac{s}{\sqrt{n}}\right)=0.95$$

- A sample of 20 female students gave a mean weight of 60kg and a standard deviation of 8 kg. Assuming normality, find the 90, 95, and 99 percent confidence intervals for the population mean weight .

$$90\%CI = \left[60 - 2.09\frac{8}{\sqrt{20}}, 60 + 2.09\frac{8}{\sqrt{20}}\right] = [56.91, 63.09]$$

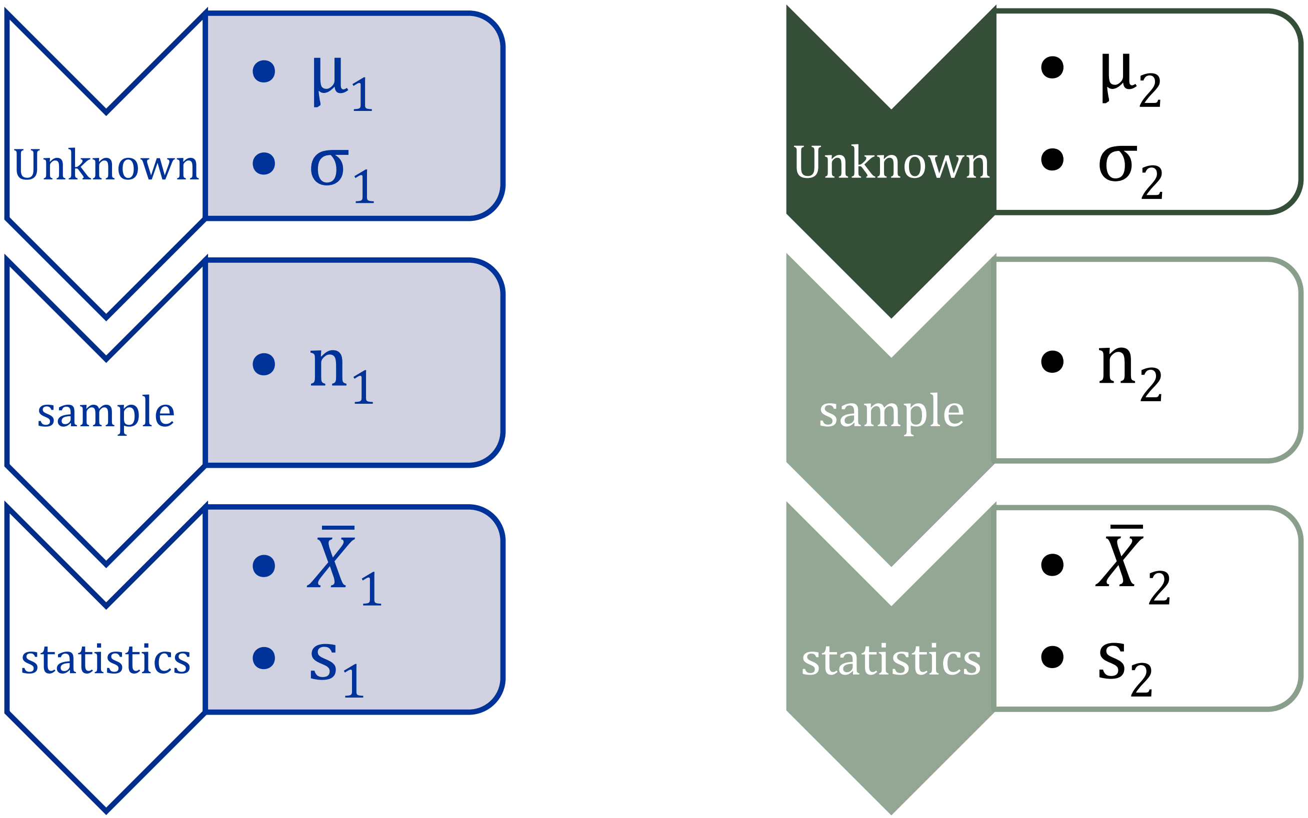
$$95\%CI = \left[60 - 2.09\frac{8}{\sqrt{20}}, 60 + 2.09\frac{8}{\sqrt{20}}\right] = [56.26, 63.74]$$

$$99\%CI = \left[60 - 2.09\frac{8}{\sqrt{20}}, 60 + 2.09\frac{8}{\sqrt{20}}\right] = [54.88, 65.12]$$

Confidence Intervals for Means Difference

Population 1

Population 2



Estimate $\mu_1 - \mu_2$ with $\bar{X}_1 - \bar{X}_2$

Confidence Intervals for Means Difference

$$(\bar{X}_1 - \bar{X}_1) \pm t_{df}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_{12}}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$$

Group 1	7	7	8	8	8	6	9	6	5
Group 2	8	10	9	6	10	8	9	7	8

	Group 1	Group 2
Mean	7.11	8.33
s	1.27	1.32
s^2	1.61	1.75

df=15.97

for $\alpha = 0.05 \longrightarrow t_{15.97} = 2.13$

$$(7.11 - 8.33) \pm 2.13\sqrt{0.18 + 0.19}$$

-1.22 ± 2.13*0.61

-1.22 ± 1.30 ⟶ [-2.52, 0.08]

CONFIDENCE INTERVALS

- Interpretation of CI for difference between two means
  - If 0 is contains by the confidence intervals, there is no significant difference between means.
  - If 0 is NOT contains by the confidence intervals, there is a significant difference between means.

Comparing Means using Confidence Intervals

http://www.biomedcentral.com/content/pdf/1471-2458-12-1013.pdf

Table 1 Living conditions of the MS-MV and the immigrant population (CASEN survey 2006)

	IMMIGRANT POPULATION 1% total sample, n = 154 431 weighted population (1877 real observations)		**MS-MV GROUP 0.67% total sample, n = 108 599 weighted population (1477 real observations)**	
	% or mean	**95% CI**	**% or mean**	**95% CI**
DEMOGRAPHICS				
Mean age**	$X = 33.41$	31.81–35.00	X = 26.13	23.41–28.26
Age categories:				
<16 years old**	13.60	11.29–16.28	45.25	39.53–51.10
16-65 years old**	79.08	75.92–81.93	47.26	41.64–52.94
>65 years old	7.32	5.33–9.97	7.49	5.31–10.46
Sex (female = 1)	45.21	41.74–48.72	51.27	47.99–55.41
Marital status:				
Single**	45.81	42.06–49.62	64.30	59.36–68.95
Married**	45.49	41.66–49.36	29.39	25.09–34.10

Confidence Interval for Frequency

- Could be computed if:
  - $n \times f > 10$, where n = sample size, f = frequency

$$\left[f - Z_{\alpha}\sqrt{\frac{f(1-f)}{n}} ; f + Z_{\alpha}\sqrt{\frac{f(1-f)}{n}} \right]$$

CONFIDENCE INTERVAL FOR FREQUENCY

- We are interested in estimating the frequency of breast cancer in women between 50 and 54 years with positive family history. In a randomized trial involving 10,000 women with positive history of breast cancer were found 400 women diagnosed with breast cancer.
- What is the 95% confidence interval associated frequently observed?

- f = 400/10000 = 0.04

$$\left[0.04 - 1.96\sqrt{\frac{0.04 \cdot 0.96}{10000}}; 0.04 + 1.96\sqrt{\frac{0.04 \cdot 0.96}{10000}}\right]$$

- [0.04-0.004; 0.04+0.004]
- [0.036; 0.044]

Confidence Intervals for Other Estimators

http://www.biomedcentral.com/content/pdf/1471-2458-12-1013.pdf

Table 3 Odds Ratio (OR) of presenting **any disability and any chronic condition or cancer**, adjusted by different sets of factors separately (CASEN survey 2006)

	ANY DISABILITY				**ANY CHRONIC CONDITION OR CANCER**			
	International immigrants		**MS-MV**		**International immigrants**		**MS-MV**	
	OR	95% CI	OR	95% CI	OR	95% CI	OR	95% CI
DEMOGRAPHICS								
Age	**1.04***	**1.02-1.06**	**1.04***	**1.02-1.06**	**1.05***	**1.02-1.08**	**1.02***	**1.01-1.04**
Sex (female = 1)	0.56	0.25-1.25	**0.39***	**0.20-0.75**	**2.78****	**1.26-6.71**	1.05	0.46-2.36

REMEMBER!

- Correct estimation of a population parameter is done with confidence intervals.
- Confidence intervals depend by the sample size and standard error.
- The confidence intervals is larger for:
  - High value of standard error
  - Small sample sizes