PROBABILITIES

Problem 1
Let A be the event that a person has normal diastolic blood pressure (DBP < 90 mmHg) and let there be the event that a person has borderline DBP reading (90 ≤ DBP ≤ 95) Suppose that P(A)= 0.7 and P(B)= 0.1.
Let C be the event that a person has DBP < 95. Compute P(C).

Problem 2
Suppose two doctors, A and B, diagnose all patients coming into a clinic for syphilis. Let the events A^+ = {doctor A makes a positive diagnosis}, B^+ = {doctor B makes a positive diagnosis}. Suppose that doctor A diagnoses 12% of all patients as positive, doctor B diagnoses 15% of all patients as positive, and both doctors diagnose 10% of all patients as positive. Suppose a patient is referred for further lab tests if either doctor A or B makes a positive diagnosis. What is the probability that patients will be referred for further lab tests?

Problem 3
We are planning a 5-year study of cataract in a population of 5000 people 60 years of age or older. We know from census data that 45% of these populations are ages 60-64, 28% are ages 65-69, 20% are ages 70-74 and 7% are age 75 or older. We also know from a study that 2.4%, 4.6%, 8.8% and 15.3% of the people in those respective age groups will develop cataract over the next 5 years. What percentage of our population will develop cataract over 5 years and how many cataracts does this percentage represent?

Problem 4
Assume that a mother, a father and two children form a family.
Consider the following events:

A_1 = {mother has hepatitis}
A_2 = {father has hepatitis}
A_3 = {first child has hepatitis}
A_4 = {second child has hepatitis}
B = {at least one child has hepatitis}
C = {at least one parent has hepatitis}
D = {at least one person has hepatitis}

a. What does $A_1 \cup A_2$ mean?
b. What does $A_1 \cap A_2$ mean?
c. Are A_3 and A_4 mutually exclusive?
d. What does $A_3 \cup B$ mean?
e. What does $A_3 \cap B$ mean?
f. Express C in terms of A_1, A_2, A_3, A_4.
g. Express D in term s of B and C.
h. What does $\overline{A_1}$ mean?
i. What does $\overline{A_2}$ mean?
j. Represent $\overline{C}$ in terms of A_1, A_2, A_3, A_4.
k. Represent $\overline{D}$ in terms of B and C.

Problem 5
A drug company is developing a new pregnancy-test kit for use on an outpatient basis. The company uses the pregnancy test on 150 women who are known to be pregnant, of whom 130 are positive, using the test. The company uses the pregnancy test on 150 other women who are known to not be pregnant, of whom 145 are negative, using the test.

a. What is the sensitivity of the test?
b. What is the specificity of the test?
c. The company anticipates that 10% of the women who will use the pregnancy test kit will be accurately identified as pregnant.
d. What is the predictive value positive for the test?

Problem 6

We can classify infants as low birth weight if they have a birth weight $\leq$ 2500 g and as normal birth weight if they have > 2500 g. Infants can also be classified by length of gestation in the following four categories: < 20 weeks, 20 - 27 weeks, 28-36 weeks, > 36 weeks. Assume that the probabilities of the different periods of gestation are as given in the next table:

Length of gestation	**Probability**
< 20 weeks	0.0005
20 - 27 weeks	0.0060
28 - 36 weeks	0.0785
> 36 weeks	0.9098

Also assume that the probability of being low birth weight given that the length of gestation is < 20 weeks is 0.540, the probability of being low birth weight given that the length of gestation is 20 - 27 weeks is 0.813, the probability of being low birth weight given that the length of gestation is 28 - 36 weeks is 0.0379 and the probability of being low birth weight given that the length of gestation is > 36 weeks is 0.035.

a. What is the probability of having a low birth weight infant?
b. Show that the events (length of gestation $\leq$ 27 weeks) and (low birth weight) are not independent.
c. What is the probability of having a length of gestation $\leq$ 36 weeks given that a child is low birth weight?