Probabilities

Problem 1

Let A be the event that a person has normal diastolic blood pressure (DBP < 90 mmHg) and let there be the event that a person has borderline DBP reading (90 ≤ DBP ≤ 95) Suppose that Pr(A)= 0.7 and Pr(B)= 0.1.

Let C be the event that a person has DBP < 95. Compute Pr(C).

Solution

If the outcomes A and B are two events that cannot both happen at the same time, then

Pr(A OR B) = Pr(A)+Pr(B)

Pr(C) = 0.7 + 0.1 = 0.8

Two events A and B are mutually exclusive if they cannot both happen at the same time.

Problem 2

Suppose two doctors, A and B, diagnose all patients coming into a clinic for syphilis. Let the events A^+ = {doctor A makes a positive diagnosis}, B^+ = {doctor B makes a positive diagnosis}. Suppose that doctor A diagnoses 12% of all patients as positive, doctor B diagnoses 15% of all patients as positive, and both doctors diagnose 10% of all patients as positive. Suppose a patient is referred for further lab tests if either doctor A or B makes a positive diagnosis. What is the probability that patients will be referred for further lab tests?

Solution

$Pr(A^+) = 0.12$

$Pr(B^+) = 0.15$

Pr(A AND B) = 0.10

One patient will be referred for further lab tests, if either doctor A or B makes a positive diagnosis. That means A^+ or B^+ event happens, or $A^+ \cup B^+$ occurs. Therefore, from the addition law of probability,

$$Pr(A^+ \cup B^+) = Pr(A^+) + Pr(B^+) - Pr(A^+ \cap B^+) = 0.12 + 0.15 - 0.10 = 0.17$$

Thus, 17% of all patients will be referred for further lab tests.

Problem 3

We are planning a 5-year study of cataract in a population of 5000 people 60 years of age or older. We know from census data that 45% of these populations are ages 60-64, 28% are ages 65-69, 20% are ages 70-74 and 7% are age 75 or older. We also know from a study that 2.4%, 4.6%, 8.8% and 15.3% of the people in those respective age groups will develop cataract over the next 5 years. What percentage of our population will develop cataract over 5 years and how many cataracts does this percentage represent?

Solution:

Let A_1, A_2, A_3, A_4 be the events:

- A_1 = {ages 60-64}
- A2 = {ages 65-69}

- A3 ={ages 70-74}
- A4 ={ages 75+}

These events are mutually exclusive and exhaustive, since exactly one event occur for each person in our population.

We also know that:

- $Pr(A_1) = 0.45$
- $Pr(A_2) = 0.28$
- $Pr(A_3) = 0.2$
- $Pr(A_4) = 0.07$
- $Pr(B|A_1) = 0.024$
- $Pr(B|A_2) =0.046$
- $Pr(B|A_3) = 0.088$
- $Pr(B|A_4) = 0.153$

Let B be the event that a person 60 years of age or older develops cataract over the next 5 years. Using the total probability rule, we have:

$$Pr(B)=Pr(B|A_1)\times Pr(A_1)+Pr(B|A_2)\times Pr(A_2)+Pr(B|A_3)\times Pr(A_3)+Pr(B|A_4)\times Pr(A_4)$$

$$Pr(B) = 0.024\times 0.45 + 0.046\times 0.28 + 0.088\times 0.2 + 0.153\times 0.07 = 0.052$$

Thus, 5.2% of our population will develop cataract over the next 5 years, which represents a total of 5000×0.52 = 260 persons with cataract.

Problem 4

Assume that a mother, a father and two children form a family.

Consider the following events:

A_1 = {mother has hepatitis}

A_2 = {father has hepatitis}

A_3 = {first child has hepatitis}

A_4 = {second child has hepatitis}

B = {at least one child has hepatitis}

C = {at least one parent has hepatitis}

D = {at least one person has hepatitis}

a. What does $A_1 \cup A_2$ mean?

b. What does $A_1 \cap A_2$ mean?

c. Are A_3 and A_4 mutually exclusive?

d. What does $A_3 \cup B$ mean?

e. What does $A_3 \cap B$ mean?

f. Express C in terms of A_1, A_2, A_3, A_4.

g. Express D in term s of B and C.

h. What does $\overline{A_1}$ mean?

i. What does $\overline{A_2}$ mean?

j. Represent $\overline{C}$ in terms of A_1, A_2, A_3, A_4.

k. Represent $\overline{D}$ in terms of B and C.

Solution:

a. Mother OR father has hepatitis / At least one parent has hepatitis.
b. Mother AND father has hepatitis / Both parents have hepatitis.
c. No, because both children could have hepatitis in the same time.
d. At least one child has hepatitis.
e. The first child is the child with hepatitis.
f. $C = A_1$ OR A_2
g. D = B OR C
h. The mother does not have hepatitis.
i. The father does not have hepatitis.
j. NonC = nonA_1 AND nonA_2
k. NonD = nonB AND nonC

Problem 5

A drug company is developing a new pregnancy-test kit for use on an outpatient basis. The company uses the pregnancy test on 150 women who are known to be pregnant, of whom 130 are positive, using the test. The company uses the pregnancy test on 150 other women who are known to not be pregnant, of whom 145 are negative, using the test.

a. What is the sensitivity of the test?
b. What is the specificity of the test?
c. What is the predictive value positive for the test?

Solution:

Contingency table

	Pregnancy +	Pregnancy -	Total
Pregnancy test +	130	5	135
Pregnancy test -	20	145	165
Total	**150**	**150**	**300**

a. Se = 130/150 = 0.8667
b. Sp = 145/150 = 0.9667
c. PPV = 130/135 = 0.9630