HYPOTHESIS TESTING
PARAMETRIC TESTS: TESTS ON MEANS

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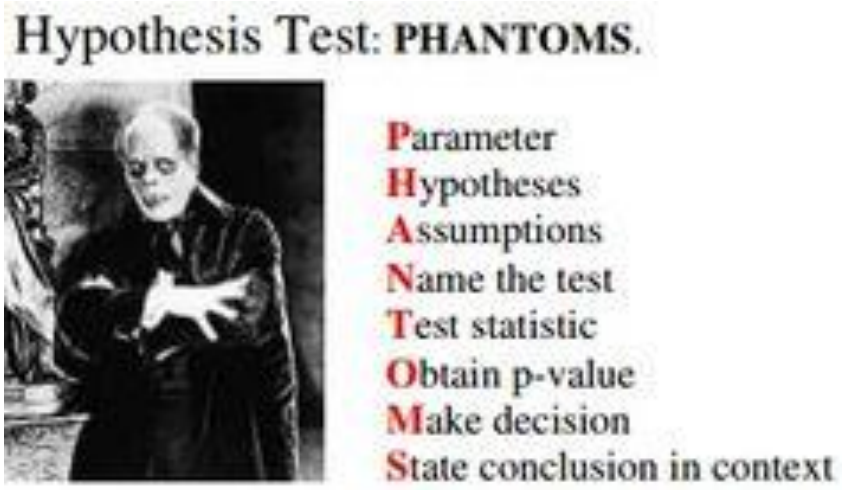


OBJECTIVES

- Significance level vs. p-value
- Hypothesis testing via confidence intervals
- Hypothesis testing on a single mean
- Comparisons of means on two samples (Student t-tests):
  - Independent sample tests
  - Dependent sample tests
- Comparisons of more than two means: ANOVA test

P VALUES vs. CONFIDENCE INTERVALS

- A **P** value measures the strength of evidence against the null hypothesis.
- A **P** value is the probability of getting a result as, or more, extreme if the null hypothesis were true.
- It is easy to compare results across studies using **P** values
- **P** values are measures of **statistical significance**
- Confidence intervals give a plausible range of values in clinically interpretable units
- Confidence intervals enable easy assessment of **clinical significance**

P VALUES vs. CONFIDENCE INTERVALS

Statistical significance can be obtained from a confidence interval as well as a hypothesis test

&

Confidence intervals convey more information than p-values

- For this reason, most medical journals now prefer that results be presented with confidence intervals rather than p-values or both them.

- If the NULL VALUE for a statistical hypothesis test using a significance level (alpha) = 0.05 is contained within the 95% confidence interval, we can conclude that there is NO statistical significance at alpha = 0.05 without doing the hypothesis test

P VALUES vs. CONFIDENCE INTERVALS

- For a 95% confidence interval for a difference between two values [-7.7 to 2.1]

- The 95% CI includes 0, so there is no statistically significant difference between the values. In addition, we have information about the precision of our estimate of the difference, which cannot be obtained from P-values alone.

!!! This is a relatively wide confidence interval maybe because the sample size is small

Relation of Confidence Intervals with Hypothesis Testing

- A general purpose approach to constructing confidence intervals is to define a $100(1-\alpha)\%$ confidence interval to consist of all those values θ_0 for which a test of the hypothesis $\theta=\theta_0$ is not rejected at a significance level of $100\alpha\%$.
  - Such an approach may not always be available since it presupposes the practical availability of an appropriate significance test.
  - Naturally, any assumptions required for the significance test would carry over to the confidence intervals.

Relation of Confidence Intervals with Hypothesis Testing

- It may be convenient to make the general correspondence that parameter values within a confidence interval are equivalent to those values that would not be rejected by an hypothesis test, but this would be dangerous.

- In many instances the confidence intervals that are quoted are only approximately valid, perhaps derived from "plus or minus twice the standard error", and the implications of this for the supposedly corresponding hypothesis tests are usually unknown.

HYPOTHESIS TESTING BY EXAMPLE

- **Null Hypothesis (H_0):**
  - There is no difference between groups
  - There is no relationship between the independent and dependent variable(s).

- **Alternative hypothesis:**
  - There is a difference between groups
  - There is a relationship between the independent nd dependent variable(s).

BMC Cardiovasc Disord. 2013 Nov 19;13(1):106. [Epub ahead of print]

Copenhagen study of overweight patients with coronary artery disease undergoing low energy diet or interval training: the randomized CUT-IT trial protocol.

Pedersen LR, Olsen RH, Frederiksen M, Astrup A, Chabanova E, Hasbak P, Holst JJ, Kjær A, Newman JW, Walzem R, Wisløff U, Sajadieh A, Haugaard SB, Prescott E.

The primary endpoint of the study is change in coronary flow reserve after the first 12 weeks' intervention.

The participants were consecutively enrolled during the inclusion period and randomized (1:1) into two groups:

1. 12 weeks of AIT (aerobic interval training) three times a week, followed by 40 weeks'AIT twice weekly.
2. 8–10 weeks' LED (low energy diet) followed by 2–4 weeks of transition to a high protein/low glycemic index diet and 40 weeks of weight loss maintenance and AIT twice weekly.

Null Hypothesis: The coronary flow reserve is not significantly different in AIT group compared with LED group.

Alternative hypothesis: The coronary flow reserve is significantly different in AIT group compared with LED group.

HYPOTHESIS TESTING BY EXAMPLE

Null and alternative hypotheses are either non-directional (two-tailed) or directional (one-tailed):

Non-directional (two-tail):

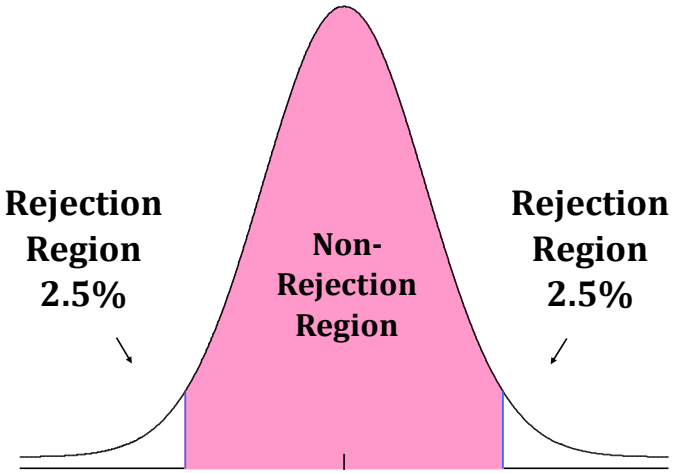
H_0: Coronary flow reserved AIT = Coronary flow reserved LED

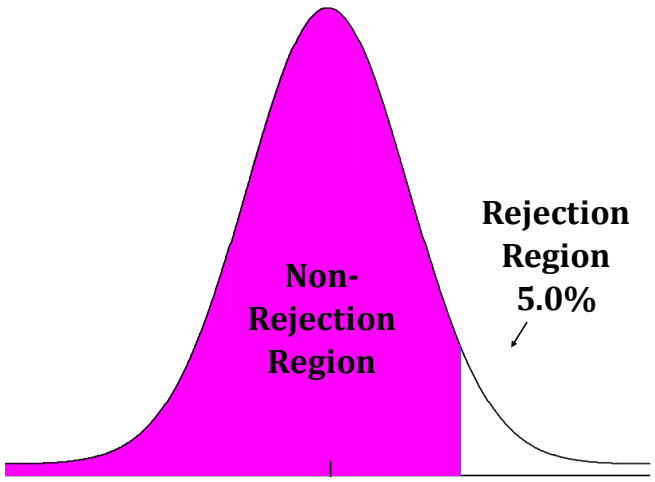
$H_{1/a}$: Coronary flow reserved AIT ≠ Coronary flow reserved LED

Directional (one-tail):

H_0: Coronary flow reserved AIT ≤ Coronary flow reserved LED or H_0: Coronary flow reserved AIT ≥ Coronary flow reserved LED

$H_{1/a}$: Coronary flow reserved AIT > Coronary flow reserved LED or $H_{1/a}$: Coronary flow reserved AIT < Coronary flow reserved LED





HYPOTHESIS TESTING BY EXAMPLE

- Alpha (α) is the level of significance in hypothesis testing
- Alpha is a probability specified before the test is performed.
- **Alpha is the probability of rejecting the null hypothesis when it is true.**
- By convention, typical values of alpha specified in medical research are **0.05** and **0.01**.
- Alphas have corresponding **critical values**, the same ones used to calculate confidence intervals – 0.05/1.96, 0.01/2.575

HYPOTHESIS TESTING BY EXAMPLE

- **Beta (β) is the probability of accepting the null hypothesis when it is false.**
- Typical values for beta are 0.10 to 0.20
- Beta is directly related to the power of a statistical test:
  - Power is the probability of correctly rejecting the null hypothesis when it is false. Power = 1 - Beta
  - A type II error occurs when a false null hypothesis is accepted.

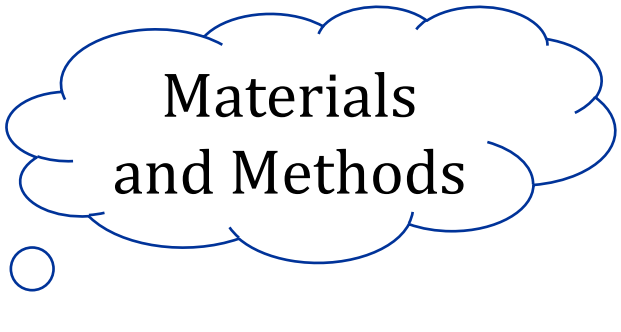
P-VALUES

- Are the actual probabilities calculated from a statistical test, and are compared against alpha to determine whether to reject the null hypothesis or not.
- Example:
  - alpha = 0.05; calculated p-value = 0.008 ➔ reject null hypothesis
  - alpha = 0.05; calculated p-value = 0.110 ➔ fail to reject null hypothesis

A type I error occurs when a true null hypothesis is rejected.

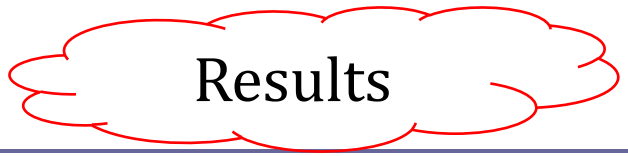
		True State of Nature	
		H_0 True	H_0 False
Findings	H_0 True	Correct	**Type II Error (β)**
	H_0 False	**Type I Error (α)**	Correct

SIGNIFICANCE LEVEL VS. p-VALUE



- Significance level (α) = property of a statistical procedure and takes a fixed value.
  - Usually take a value of 0.05

- p-value = random variable whose value depends upon the composition of the individual sample



SIGNIFICANCE LEVEL VS. p-VALUE

http://www.ncbi.nlm.nih.gov/pmc/articles/PMC4129321/

n Res

Statistical Analysis

Sample size in our study was calculated by using method described by S. Freestone et al., for sample size estimation for short term trails of antihypertensive drugs [6].The power of our study was 90% at 0.05% level of significance. Considering the loss to follow up, total 90 patients were enrolled in the study. Qualitative data such as sex, patient's habits were analyzed by using chi-square test. Data on adverse effects was analysed using z-test for difference between two proportions. Quantitative data was analysed using z-test for difference between means. p-value < 0.05 was taken as significant. p-value < 0.001 was taken as highly significant, p-value >0.05 was considered insignificant.

lin Diagn Res

Effect of nebivolol and atenolol on Heart rate

S. No	Parameters	Nebivolol	Atenolol	p-value
1	Before treatment	78.05±5.839	76.55±5.33	p>0.05
2	After treatment	63.53±3.86	59.0±3.271	—
3	Mean reduction in heart rate	14.51±4.69	17.55±5.06	p<0.05
4	p-Value	p<0.001	p<0.001	

PARAMETRIC & NON-PARAMETRIC TEST

	Parametric	Non-Parametric
Assumed distribution	Normal	Any
Assumed variance	Homogenous	Any
Type of data	Ratio or Interval	Ordinal or Nominal
Central measure	Mean	Median
Dispersion measure	Standard deviation	(Q1; Q3)

	Parametric	Non-Parametric
2 independent groups	Independent t-test	Mann-Whitney test
2 dependent groups	Paired t-test	Wilcoxon test
> 2 groups	ANOVA	Kruskal-Wallis test Friedman's ANOVA
Correlation	Pearson	Spearman, Kendall, etc.
…	…	…

HYPOTHESIS

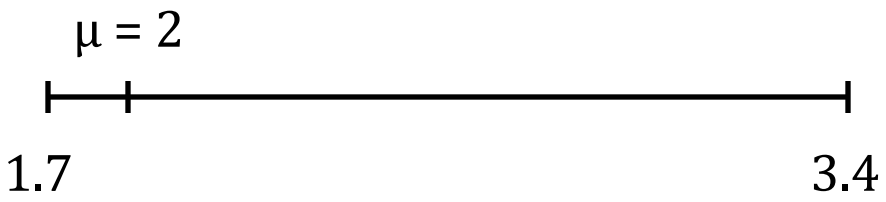
Null	H_0	Not significantly different ('=' symbol)
Alternative	H_A/H_1	Significantly different ■ two-tail: '≠' symbol ■ one-tail: '<' OR '>' symbol

HYPOTHESIS TESTING VIA CI

- n=33, m = 2.6, s = 1.2, SE = 0.21
- 95% CI for the average number exams failed by medical students in the first year of study is (1.7, 3.4). Based on this confidence interval, do these data support the hypothesis that medical students on average failed on average more than 2 exams?

H_0	$\mu = 2$	Medical student in first year of study failed 2 exams, on average
H_A	$\mu > 2$	Medical student in first year of study failed more than 2 exams, on average

Always about population parameter, never about population statistics



HYPOTHESIS TESTING VIA CI

- n=33, m = 2.6, s = 1.2, SE = 0.21
- P(observed or more extreme outcome|H_0 true)
- P(m>2.6|H_0: $\mu = 2$)
- Test statistic: Z = (2.6-2)/0.21 = 2.8571
- p-value = P(Z>2.8571) = 0.0021
  - p-value = the probability of observing data at least as favorable to the alternative hypothesis as our current data set, if the null hypothesis was true.
  - If p-value < 0.05 we say that it would be very unlikely to observe the data if the null hypothesis were true, and hence reject H_0.
  - If the p-value > 0.05 we say that it is likely to observe the data even if the null hypothesis were true, and hence fail to reject H_0.

HYPOTHESIS TESTING on MEANS

HYPOTHESIS TESTING ON A SINGLE MEAN

1. Hypotheses: H_0: μ = null value & H_A: $\mu \neq$ null value
2. Calculate the point estimator
3. Check conditions:
   - Independence: observations are independent by each others
   - Sample size: $n \geq 30$
4. Draw sampling distribution, shade p-value, calculate test statistic: $Z = (\bar{x} - \mu)/(s/\sqrt{n})$
5. Make a decision:
   - p-value $< \alpha \rightarrow$ reject H_0 (data provide convincing evidence for H_A)
   - p-value $> \alpha \rightarrow$ fail to reject H_0 (data do not provide convincing evidence for H_A)

INDEPENDENT SAMPLES: ARE TWO MEANS THE SAME?

Total sample size	Subgroup sample size	Equal Variances	Unequal Variances
Large size (n>50 or n>100) or σ's known	~ equal very different	Z-test	Rank-sum test
Small size	~ equal very different	t-test	Rank-sum test

Assumptions:

❶ the observations are independent from each other;

❷ the samples are drawn from a normal distribution (use a Rank-test when this assumption is violated);

❸ standard deviation of samples are not statistically different by each other (apply an unequal variance form of the means test or a rank test).

Z and T Tests to Compare a Sample Mean with a Population Mean

Z test

When? Population standard deviation is known OR n > 50 (100)

Hypotheses:

- $m = \mu\ (H_0)$ vs. $m \neq \mu\ (H_1)$

Significance level ($\alpha = 0.05$) → critical value with n-1 df (degree of freedom)

Test statistics:

$z = (m-\mu)/(\sigma/\sqrt{n})$ where σ = standard deviation, n = sample size

t-test

When? Unknown standard deviation OR n < 50 (100)

Hypotheses:

- $\mu_1 = \mu_2\ (H_0)$ vs. $\mu_1 \neq \mu_2\ (H_1)$

Significance level ($\alpha = 0.05$) → critical value with n-1 df (degree of freedom)

Test statistics:

$t = (m-\mu)/(s/\sqrt{n})$ where s = standard deviation, n = sample size

Z and T Tests to Compare a Sample Mean with a Population Mean – Problem 1

Z-test

- $\mu = 0$ (H_0) vs $\mu \neq 0$ (H_1)
- $\alpha = 0.05$
- $\sigma = 1.75$
- $n = 15$
- $m = 3.87$
- $Z_{crit} = 1.96$

Z statistic = ?

Conclusion ?

T-test

- $\mu = 0$ (H_0) vs $\mu \neq 0$ (H_1)
- $\alpha = 0.05$
- $s = 2.50$
- $n = 15$
- $m = 3.87$
- $t_{crit} = 2.145$

t statistic = ?

Conclusion ?

Z AND T TESTS TO COMPARE TWO SAMPLE MEANS

Z test

When? Population standard deviation is knows OR n > 50 (100)

Hypotheses:

- $\mu_1 = \mu_2$ (H_0) vs. $\mu_1 \neq \mu_2$ (H_1)

Significance level ($\alpha = 0.05$) → critical value with n_1+n_2-1 df (degree of freedom)

Test statistics:

$z = (m_1-m_2)/\sigma_d$, where σ_d = population standard error ($\sigma_d = \sigma*sqrt(1/n_1+1/n_2)$)

t-test

When? Unknown standard deviation OR n < 50 (100)

Hypotheses:

- $\mu_1 = \mu_2$ (H_0) vs. $\mu_1 \neq \mu_2$ (H_1)

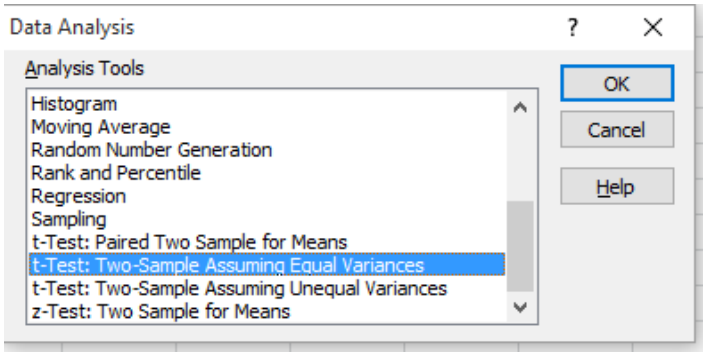
Significance level ($\alpha = 0.05$) → critical value with n_1+n_2-1 df (degree of freedom)

Test statistics:

t-statistics = $(m_1-m_2)/s_d$, where s_d = standard error

$$s_d = \sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\cdot\left[\frac{(n_1-1)\cdot {s_1}^2+(n_2-1)\cdot {s_2}^2}{n_1+n_2-2}\right]}$$

INDEPENDENT SAMPLES T-TEST



Are the variances statistically different?

t-test assuming unequal variances

t-test assuming equal variances

T TESTS TO COMPARE TWO SAMPLE MEANS

Age and prostate cancer t-test

- Negative biopsy: n_1=206, m_1=66.59 years old, s_1=8.21
- Positive biopsy: n_2=95, m_2=67.14 years old, s_2=7.88
- σ = 8.10 (n=301)
- $\alpha = 0.05 \rightarrow t_{critic} = 1.96$
- sd = sqrt((1/206+1/95)×((205*8.21^2+94*7.88^2)/(206+95-2))) = 0.3531
- t = (m_1-m_2)/sd = (66.59-67.14)/0.3531 = -1.5576 (p-value = 0.120)
- $-1.96 \leq -1.5576 \leq 1.96 \rightarrow$ we failed to reject the H_0 (The mean age of subjects with positive biopsy is not significantly different by the mean age of subjects with negative biopsy)
- For samples > 100 the difference between Z and t-statistic is negligible while the p-values are identical

T TESTS TO COMPARE TWO SAMPLE MEANS

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- $-1.96 \leq -1.5576 \leq 1.96 \rightarrow$ we failed to reject the H_0 (The mean age of subjects with positive biopsy is not significantly different by the mean age of subjects with negative biopsy)
- For samples > 100 the difference between Z and t-statistic is negligible while the p-values are identical

STUDENT T-TEST FOR COMPARING TWO MEANS (UNKNOWN AND EQUAL VARIANCES)

- **Null hypothesis**: Means difference of the two populations is not significantly different by zero.
- **Alternative hypothesis** for two-tailed test: Means difference of the two populations is significantly different by equal.
- **Assumptions:**
  - The variables in the two samples are normal distributed
  - The variances are equal
- If these two assumptions are not satisfied the test loss its validity.
- If the variances of populations are known the Z test is applied (is most powerful)

STUDENT T-TEST FOR COMPARING TWO MEANS (UNKNOWN AND EQUAL VARIANCES)

- **Degree of freedom (df):**
  - $df = n_1 + n_2 - 2$
- **Significance level**: $\alpha = 0.05$
- **Critical region** for two-tailed test

$$\left(-\infty;-t_{n_1+n_2-2;\frac{\alpha}{2}}\right] \cup \left[t_{n_1+n_2-2;\frac{\alpha}{2}};+\infty\right)$$

- **Statistics**

$$t = \frac{m_1 - m_2}{\sqrt{s\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$s = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

STUDENT T-TEST FOR COMPARING TWO MEANS (UNKNOWN AND EQUAL VARIANCES)

We want to study whether there is a significant difference between the amount of blood uric acid in women from urban and rural. In a sample of 16 women aged between 30 and 50 years in urban areas, average uric acid was 5 mg/100 ml, with a variance of 2 mg/100 ml. An average equal to 4 mg/100 ml with a variance of 2 mg/100 ml was obtained on a sample of 16 women aged 30 to 50 years in rural areas.

- **Data**:
  - $n_1 = 16$; $n_2 = 16$
  - $m_1 = 5$; $m_2 = 4$
  - $s^2 = 2$
- **Null hypothesis**: The mean of uric acid in women from urban is not significantly different by the mean of acid in women from rural.
- **Alternative hypothesis** (two-tail test): The mean of uric acid in women from urban is not significantly different by the mean of acid in women from rural.

STUDENT T-TEST FOR COMPARING TWO MEANS (UNKNOWN AND EQUAL VARIANCES)

- **Degree of freedom**: $df = n_1+n_2-2 = 16+16-2=30$
- **Significance level**: $\alpha = 0.05$.
- **Critical region** for bilateral test: $(-\infty;-2.04]\cup[2.04;+\infty)$

$$s=\sqrt{\frac{(n_1-1)\cdot s_1^2+(n_2-1)\cdot s_2^2}{n_1+n_2-2}}=\sqrt{\frac{(16-1)\cdot 2+(16-1)\cdot 2}{16+16-2}}=\sqrt{\frac{60}{30}}=1.41$$

$$t=\frac{m_1-m_2}{\sqrt{s\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}=\frac{5-4}{\sqrt{1.41\cdot\left(\frac{1}{16}+\frac{1}{16}\right)}}=\frac{1}{\sqrt{1.41\cdot 0.25}}=\frac{1}{\sqrt{0.3525}}=\frac{1}{0.5937}=1.68$$

Conclusion:

- Statistical: The null hypothesis is fail to be rejected since the statistics did not belongs to the critical region.
- Clinical: The serum level of uric acid is not significantly different in women from rural compared to those from urban areas.

STUDENT T-TEST FOR COMPARING TWO MEANS (UNKNOWN AND EQUAL VARIANCES)

spark.rstudio.com/minebocek/dist_calc/

Distribution Calculator

Distribution:

t

Degrees of freedom

1 30 50

Model:

P(X < a or X > b)

Find Area:

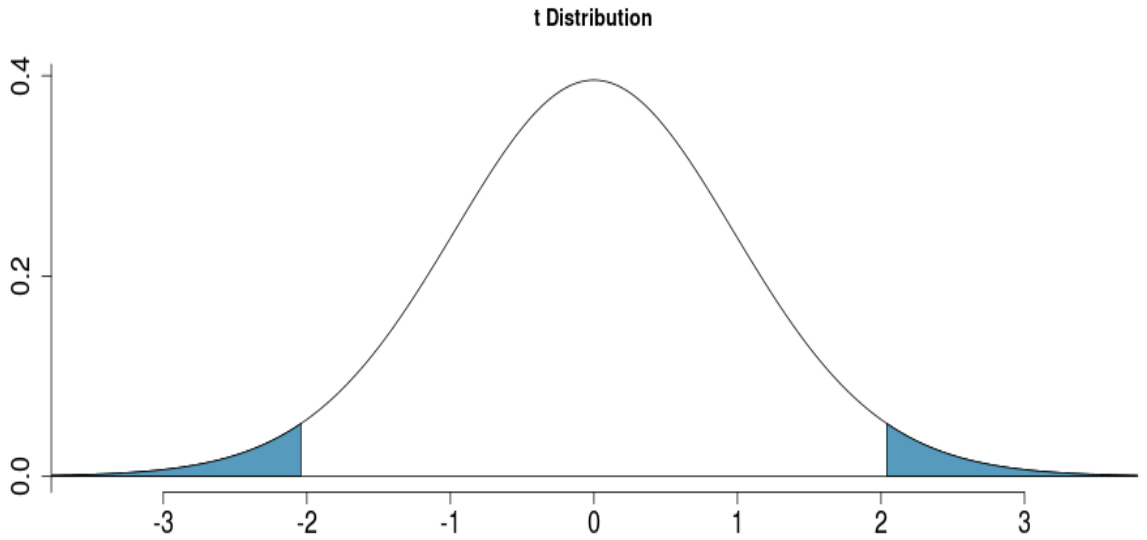
Both Tails

a

-6 -2.04 6

b

-6 2.04 6



P(X < -2.04 or X > 2.04) = 0.0502

of asphalt binder in low, intermediate and high service temperatures.

4.2.1. The mean hypothesis test method

Due to test conditions, the hypothesis test of two populations with unknown variances and equal number of samples is used in this paper. The steps of this method are [29]:

1. $H_0: \mu_1 - \mu_2 = 0$
2. $H_1: \mu_1 - \mu_2 > 0$ or $H_1: \mu_1 - \mu_2 < 0$
3. $T_{f,n_1+n_2-2} = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

where H_0 is the null hypothesis, H_1 is the alternative hypothesis, T is the test statistics, $\overline{X}_1$ and $\overline{X}_2$ are the mean value measurements of two population samples, n_1 and n_2 are the numbers of each sample that has the equal quantity, and s_p^2 is the combined variance of the two samples which can be calculated as:

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

where s_1^2 and s_1^2 are the variances of two samples.

4. T has the t-distribution with $n-1$ degree of freedom, which α is the significant level.
5. The acceptable area is defined as:

$$T < t_{1-\alpha}$$

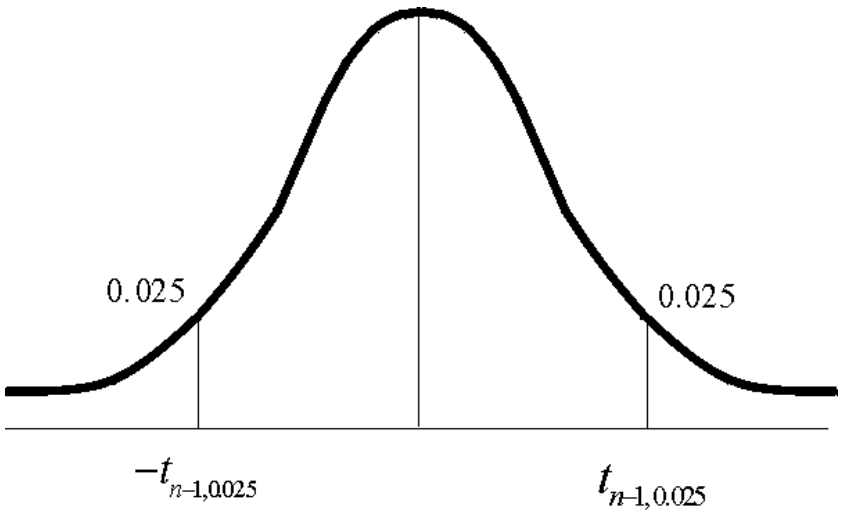
PAIRED SAMPLES
STUDENT T-TEST FOR COMPARING MEANS

- **Aim:** comparing the means of two paired samples on quantitative continuous variable (paired means the observation of the same quantitative variable before and after the action of a factor)
- **Assumptions:**
  - Individual observations from the first sample corresponds to a pair in the second sample
  - The differences between pairs of values are normally distributed.
- **Null hypothesis**: The difference of means is not significantly different by zero.
- **Alternative hypothesis** (two-tail): The difference of means is significantly different by zero.

STUDENT (T) FOR COMPARING MEANS OF PAIRED SAMPLES

- **Degrees of freedom (df):** $df = n - 1$
- **Significance level:** $\alpha = 0.05$
- **Critical region:**

$$(-\infty;-t_{n-1;\frac{\alpha}{2}}] \cup [t_{n-1;\frac{\alpha}{2}};+\infty)$$



- **Statistics**

$$t = \frac{\bar{d}}{\frac{s}{\sqrt{n}}}$$

The mean of the difference of paired data

$$\bar{d} = \frac{(d_1 + d_2 + \ldots + d_n)}{n}$$

d_1 = the difference between paired data for the first subject

- s = standard deviation of the differences
- n = sample size

PAIRED STUDENT T-TEST

Paired *t* Test

Denote the test statistic $\bar{d}/(s_d/\sqrt{n})$ by t, where s_d is the sample standard deviation of the observed differences:

$$s_d = \sqrt{\left[\sum_{i=1}^{n} d_i^2 - \left(\sum_{i=1}^{n} d_i\right)^2 \Big/ n\right] \Big/ (n-1)}$$

n = number of matched pairs

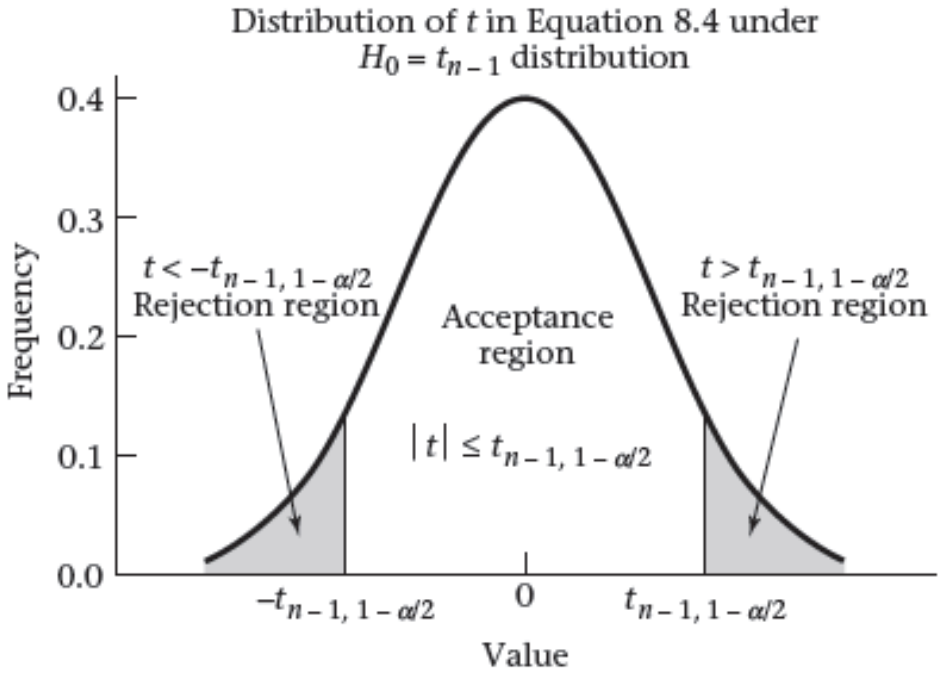
If $t > t_{n-1,1-\alpha/2}$ or $t < -t_{n-1,1-\alpha/2}$

then H_0 is rejected.

If $-t_{n-1,1-\alpha/2} \le t \le t_{n-1,1-\alpha/2}$

then H_0 is accepted. The acceptance and
in Figure 8.1.

Figure 8.1 Acceptance and rejection regions for the paired *t* test



PAIRED STUDENT T-TEST

SBP levels (mm Hg) in 10 women while not using (baseline) and while using (follow-up) OCs

i	SBP level while not using OCs (x_{i1})	SBP level while using OCs (x_{i2})	d_i^*
1	115	128	13
2	112	115	3
3	107	106	−1
4	119	128	9
5	115	122	7
6	138	145	7
7	126	132	6
8	105	109	4
9	104	102	−2
10	115	117	2

*$d_i = x_{i2} - x_{i1}$

Assume that the SBP of the ith woman is normally distributed at baseline with mean μ_i and variance σ^2 and at follow-up with mean $\mu_i + \Delta$ and variance σ^2.

STUDENT T-TEST FOR COMPARING MEANS OF PAIRED SAMPLES

- **Null hypothesis**: There is no significant difference in systolic blood pressure before and after using of oral contraceptives.
- **Alternative hypothesis** for two-tailed test: There is a significant difference in systolic blood pressure before and after using of oral contraceptives.
- **Degrees of freedom**: df = n – 1 = 10-1 = 9
- **Significance level**: $\alpha = 0.05$
- **Critical regions** for two-tailed test: $(-\infty; -2.262] \cup [2.262; +\infty)$

STUDENT T-TEST FOR COMPARING MEANS OF PAIRED SAMPLES

$$\bar{d} = \frac{13+3-1+9+7+7+6+4-2+2}{10} = \frac{48}{10} = 4.8$$

$$s = \sqrt{\frac{67.24+3.24+33.64+(4.2)^2+2\cdot 4.84+1.44+0.64+46.24+7.84}{10-1}} = \sqrt{\frac{187.60}{9}} = \sqrt{20.84} = 4.57$$

$$t = \frac{\bar{d}}{\frac{s}{\sqrt{n}}} = \frac{4.8}{\frac{4.57}{\sqrt{9}}} = \frac{4.8}{\frac{4.57}{3}} = \frac{4.8}{1.52} = 3.15$$

Conclusion (two-sided test):

- Statistical: The null hypothesis is rejected since the statistics belongs to critical region.
- Clinical: The use of oral contraceptives is associated to a significant increase in systolic blood pressure.

THREE OR MORE MEANS: ANOVA

Are the means of k groups different?

- H_0: There are no differences among the m_i.
- H_1: A difference exists somewhere among the groups

Is the t-test appropriate?

No, because the t-test compare two groups and this approach will increase the size of the error as the number of groups is higher than 2 instead of 5%

THREE OR MORE MEANS: ANOVA

Solution: apply ANOVA (analysis of variance, one-factor ANOVA or one-way ANOVA)

ANOVA Assumptions:

❶ data are independent from each other;

❷ distribution of each group in original data is normal;

❸ the variances are not significantly different by each other

THREE OR MORE MEANS: ANOVA

- Hypotheses: H_0: There are no differences among means
- H_1: There are one or more differences somewhere among means
- Verify assumptions: ❷ normal distribution; ❸ not statistically different variances
- $\alpha = 0.05$ – df = k-1 (numerator) and df = n-k (denominator)
- F = MSM/MSE
- If $F > F_{crit}$ → reject H_0; $F < F_{crit}$ → failed to reject H_0

Source of variability	Sum of Squares			Mean of squares	
	Abb	Formula	df	Abb	Formula
Mean	SSM	$\sum^{k} n_i \cdot (m_i - m)^2$	k-1	MSM	SSM/(k-1)
Error	SSE	SST – SSM	n-k	MSE	SSE/(n-k)
Total	SST	$\sum^{n} (x_i - m)^2$	n-1	MST	SST/(n-1)

Three or More Means: ANOVA - Problem

α=0.05; df: k-1=3-1=2; n-k=301-3=298; F_{crit} = 3.03; m = 66.8; SST = 19670.3; MST = 65.57

PSA	CaP risk group	n	m	s
< 4 ng/ml	Low	89	66.1	9.1
4–10 ng/ml	Uncertain	164	66.3	7.8
>10 ng/ml	High	48	69.6	6.4

PSA=prostate-specific antigen; CaP = prostate cancer; n = sample size; m = mean; s = standard deviation

Source of variability	Sum of Squares			Mean of squares	
	Abb	Formula	df	Abb	Formula
Mean	SSM	$\sum^{k} n_i \cdot (m_i - m)^2$	k-1	MSM	SSM/(k-1)
Error	SSE	SST – SSM	n-k	MSE	SSE/(n-k)
Total	SST	$\sum^{n} (x_i - m)^2$	n-1	MST	SST/(n-1)

THREE OR MORE MEANS: ANOVA - PROBLEM

PSA	CaP risk group	n	m	s
< 4 ng/ml	Low	89	66.1	9.1
4–10 ng/ml	Uncertain	164	66.3	7.8
>10 ng/ml	High	48	69.6	6.4

PSA=prostate-specific antigen; CaP = prostate cancer; n = sample size; m = mean; s = standard deviation

$SSM = \sum^{k} n_i \cdot (m_i - m)^2 = 89*(66.1-66.8)^2 + 164*(66.3-66.8)^2 + 48*(69.6-66.8)^2 = 460.93$

$MSM = SSM/2 = 460.93/2 = 230.47$

$SSE = SST - SSM = 19670.3 - 460.93 = 19209.37$

$MSE = SSE/298 = 19209.37/298 = 64.46$

$F = MSM/MSE = 230.47/64.46 = 3.58$

Since $3.58 > 3.03 \rightarrow$ a difference among means exists

TESTS ON MEANS BY EXAMPLES

http://www.hindawi.com/journals/tswj/2013/608683/tab2/

Table 2: The values of blood pressures and heart rates prior to and after the treatment with metoprolol and nebivolol in the study

	Total (n = 60)	**Metoprolol** (n = 30)	**Nebivolol** (n = 30)	P **value**
Systolic blood pressure (mm/Hg)				
Before	152.1 ± 4.9	151.3 ± 3.6	152.8 ± 5.9	0.27**
After	136.4 ± 10.9	134.9 ± 10.5	137.8 ± 11.3	0.3**
P value	<0.001*	<0.001*	<0.001*	
Diastolic blood pressure (mm/Hg)				
Before	91.7 ± 3.9	91.1 ± 3.7	92.3 ± 4.1	0.25**
After	82.6 ± 7.1	82.4 ± 6.7	82.7 ± 7.3	0.87**
P value	<0.001*	<0.001*	<0.001*	
Heart rate (pulse/min)				
Before	75.7 ± 5.7	75.2 ± 5.7	76.2 ± 5.7	0.5**
After	70.2 ± 5.4	70.3 ± 5.8	70.1 ± 5.1	0.87**
P value	<0.001*	<0.001*	<0.001*	
Achieved targeted blood pressure (n, %)	37 (61.6)	19 (63.3)	18 (60)	0.5***

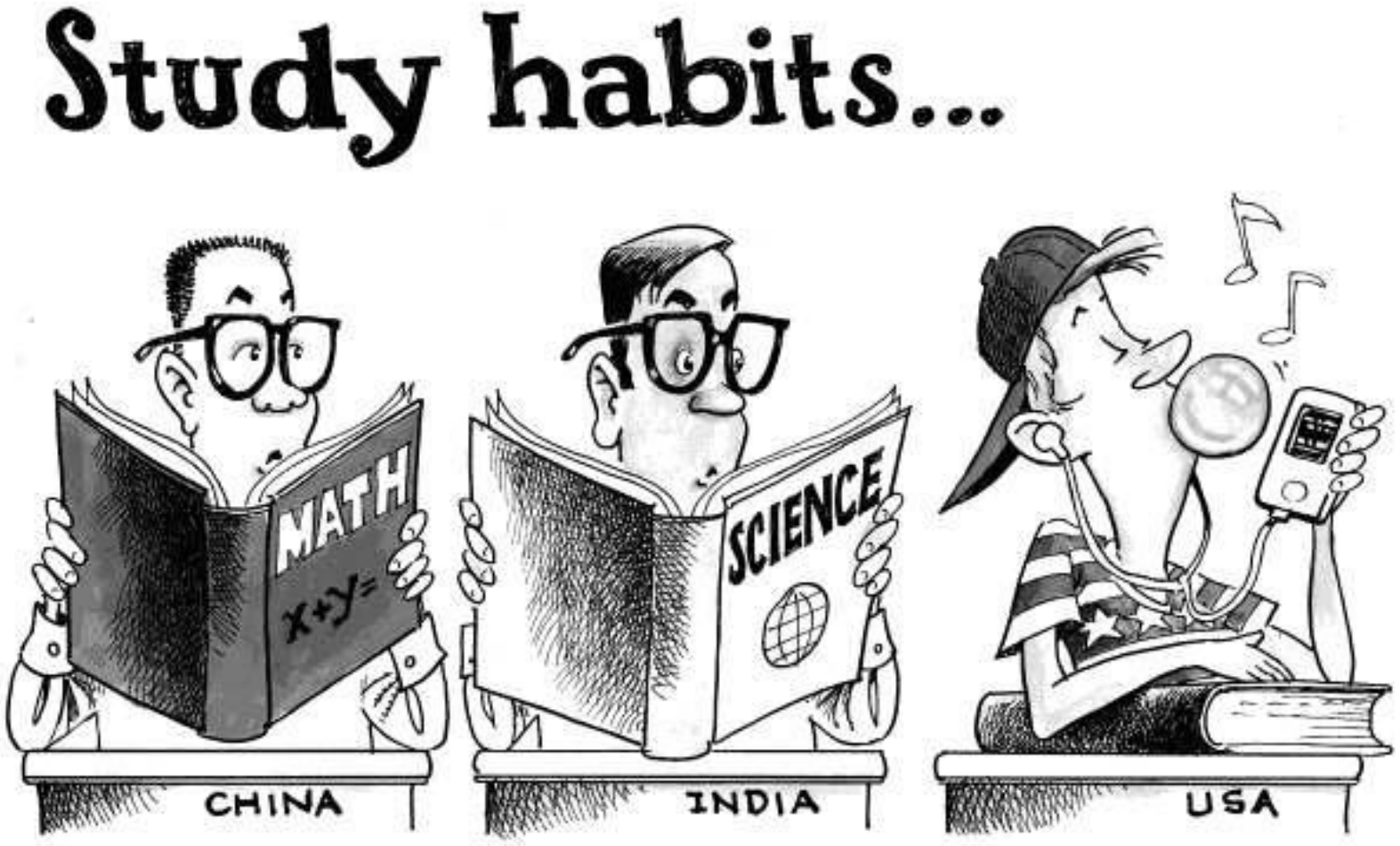
*Paired t-test; **independent samples t-test; ***chi-square test.

http://www.hindawi.com/journals/tswj/2013/608683/tab2/

Table 3: The values of blood pressures and heart rates measured during rest, exercise, and recovery periods in the study population.

	Total ($n = 60$)	**Metoprolol** ($n = 30$)	**Nebivolol** ($n = 30$)	P **value**
Systolic blood pressure (mm/Hg)				
Rest	137.4 ± 13.7	135.3 ± 13.5	139.6 ± 13.8	0.22
Stage 1	152 ± 22.5	151.7 ± 20.9	152.3 ± 24.4	0.91
Stage 2	162.7 ± 27.1	160.8 ± 27.1	164.6 ± 27.5	0.61
Stage 3	175.3 ± 29.3	174.3 ± 29.7	176.2 ± 29.7	0.84
Recovery (3 min)	164.4 ± 27.4	167.1 ± 29.8	161.7 ± 25	0.46
Recovery (5 min)	156.5 ± 27.9	158.5 ± 27	154.5 ± 29	0.58
Diastolic blood pressure (mm/Hg)				
Rest	82.6 ± 9.9	83.7 ± 10.6	81.5 ± 9.3	0.39
Stage 1	76 ± 21.2	78.6 ± 21.5	73.3 ± 20.9	0.34
Stage 2	72.9 ± 18.7	76.7 ± 16.5	68.9 ± 20.2	0.12
Stage 3	71.5 ± 19.5	77.3 ± 11.5	66.7 ± 23.4	0.07
Recovery (3 min)	77.9 ± 17	81.8 ± 16.6	74.1 ± 16.7	0.08
Recovery (5 min)	81.9 ± 12.9	83.2 ± 14.1	80.7 ± 11.8	0.45
Heart rate per minute				
Rest	71.7 ± 5.5	72.1 ± 5.3	71.2 ± 5.9	0.63
Stage 1	106.7 ± 16.4	105.2 ± 17.2	108.1 ± 15.7	0.49
Stage 2	122.8 ± 17.2	119.6 ± 16.6	126.3 ± 17.6	0.15
Stage 3	139.5 ± 18.5	140.3 ± 18.1	138.9 ± 19.2	0.82
Recovery (3 min)	118.7 ± 16.3	117.2 ± 17.7	120.1 ± 14.9	0.5
Recovery (5 min)	90.3 ± 14.5	89.4 ± 15.2	91.2 ± 14	0.63

Thank you for your attention!



Thank you!